NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSAM	**LEVEL:** 7
COURSE CODE: NUM702S	**COURSE NAME:** NUMERICAL METHODS 2
SESSION: NOVEMBER 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY – QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Attachments

None

Problem 1 [40 Marks]

1-1. Show that the formula for the best line to fit data (k, y_k) at integers k for $1 \leq k \leq n$ is $y = ax + b$, where [15]

$$a = \frac{6}{n(n^2-1)}\left[2\sum_{k=1}^{n} k y_k - (n+1)\sum_{k=1}^{n} y_k\right] \quad \text{and} \quad b = \frac{4}{n(n-1)}\left[(2n+1)\sum_{k=1}^{n} y_k - 3\sum_{k=1}^{n} k y_k\right]$$

1-2. Establish the Padé approximation $e^x \approx R_{2,2}(x) = \dfrac{12+6x+x^2}{12-6x+x^2}$ and express $R_{2,2}$ in continued fraction form. [10]

1-3. Write down the general formula of $S_f(x)$, the Fourier series of a function f that is 2π periodic, piece-wise continuous and defined on $(-\pi, \pi)$. [5]

1-4. Find the Fourier sine series for the 2π-periodic function $f(x) = x(\pi - x)$ on $(0, \pi)$. [Hint: Assume f is an odd function]. Use its Fourier representation to find the value of the infinite series

. [10=7+3]

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} + \cdots$$

Problem 2 [31 Marks]

2-1. Define $T_n(x)$, the nth degree Chebyshev polynomial of the first kind for $x \in [-1, 1]$ and show that:

(i) $T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x)$, for $k \geq 1$, with $T_0(x) = 1$, $T_1(x) = x$; [5]

(ii) T_n has n distinct zeros/roots $x_k = \cos\left(\dfrac{(2k+1)\pi}{2n}\right)$ for $0 \leq k \leq n-1$. [7]

2-2. Use the formulae in (i) of question **2-1** to find $T_2(x), T_3(x)$ and then economize $P(x) = 1 + 2x^2 + 3x^3$, once. [6]

2-3. Given the integral $\displaystyle\int_0^3 \frac{\sin(2x)}{1+x^5}dx = 0.6717578646\cdots$

2-3-1. Using the sequential trapezoidal rule, state the formula of $T(J) = R(J, 0)$ and then compute its values for $J = 0, 1, 2$. [3+10=13]

Problem 3 [29 Marks]

3-1. For an n-point Gaussian quadrature rule, the Legendre polynomials $q_n(x)$, for $x \in [-1, 1]$, can be generated by the recursion formula

$$q_n(x) = \left(\frac{2n-1}{n}\right) x q_{n-1}(x) - \left(\frac{n-1}{n}\right) q_{n-2}(x) \quad \text{for } n = 2, 3, \ldots \quad \text{and } q_0(x) = 1, q_1(x) = x.$$

3-1-1. Compute $q_2(x)$, $q_3(x)$ and determine the zeros of $q_3(x)$. [2+2+3=7]

3-1-2. Using the zeros of $q_3(x)$ as quadrature nodes, state the associated quadrature formula and determine the corresponding weights by the method of undetermined coefficients. How do you call the rule thus obtained? [2+8+2=12]

3-2. Consider the following matrix $A = \begin{bmatrix} 6 & 5 & -5 \\ 2 & 6 & -2 \\ 2 & 5 & -1 \end{bmatrix}$. Find its largest eigenvalue (in magnitude) and the corresponding eigenvector after three iterations with the initial vector $x^{(0)} = (-1, 1, 1)^T$. [10]

God bless you !!!